We have a combination of three qubits. Qubit $|\psi\rangle$ and an entangled pair of qubits $|\phi^+\rangle$, one of the Bell states:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

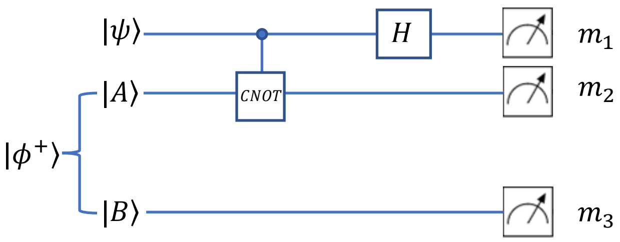
We bring qubit $|\psi\rangle$ into the definite state $0.5|0\rangle + 0.866|1\rangle$ by help of the rotation

ry(2 * pi / 3) q[0];

We call the parameters $\alpha = 0.5$ and $\beta = \sqrt{0.75} \approx 0.866$.

Note: You find this rotation at:
https://quantumcomputing.stackexchange.com/questions/16501/how-to-initialize-a-qubit-with-a-custom-state-in-qiskit-composer

We perform a CNOT from $|\psi\rangle$ to $|A\rangle$ and a Hadamard on the qubit $|\psi\rangle$ and get a superposition of all three qubits:

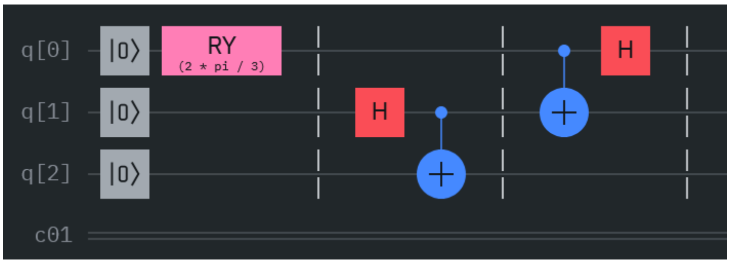


Note:

the Hadamard gate

controlled NOT gate

In the IBM composer this looks like: ① ② ③



Note: The gate RY (①) produces the state $0.5|0\rangle + 0.866|1\rangle$, giving the probability of 0.25 for $|0\rangle$ and 0.75 for $|1\rangle$.

Note: The first Hadamard and CNOT (②) produce the Bell-state $|\phi^+\rangle$ from qubits $|A\rangle$ ($q[1] = |0\rangle$) and $|B\rangle$ ($q[2] = |0\rangle$).

Let us take a closer look at the scenario.

The initial state of the qubits $|\psi\rangle$ and $|\phi^+\rangle$ we name $|\pi_0\rangle$.

In detail:

$|\pi_0\rangle = |\psi\rangle|\phi^+\rangle$ resp. $|\psi\phi^+\rangle$

The qubit $|\psi\rangle$ is in the state $\alpha|0\rangle + \beta|1\rangle$ with $\alpha = 0.5$ and $\beta = 0.8660245$.

The Bell state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

We get:

$$|\pi_0\rangle = (\alpha|0\rangle + \beta|1\rangle)|\phi^+\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle)\left(\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\right) =$$

$$\frac{\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle}{\sqrt{2}}$$

(③) We apply the $CNOT$ on $|\psi\rangle$ and get $|\pi_1\rangle$.

Note that the $CNOT$ is controlled by the first qubit and acts on the second qubit:

$$|\pi_1\rangle = \frac{\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle}{\sqrt{2}}$$

We rewrite:

$$|\pi_1\rangle = \frac{|0\rangle(\alpha|00\rangle + \alpha|11\rangle) + |1\rangle(\beta|01\rangle + \beta|10)}{\sqrt{2}}$$

(③) We apply the Hadamard on $|\psi\rangle$:

$$H|\psi\rangle = \frac{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)(\alpha|00\rangle + \alpha|11\rangle) + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)(\beta|01\rangle + \beta|10)}{\sqrt{2}} =$$

$$\frac{(|0\rangle + |1\rangle)(\alpha|00\rangle + \alpha|11\rangle) + (|0\rangle - |1\rangle)(\beta|01\rangle + \beta|10)}{2} =$$

$$\frac{\alpha|000\rangle + \alpha|011\rangle + \alpha|100\rangle + \alpha|111\rangle + \beta|001\rangle + \beta|010\rangle - \beta|101\rangle - \beta|110\rangle}{2} =$$

$$\frac{\alpha|000\rangle + \beta|001\rangle + \beta|010\rangle + \alpha|011\rangle + \alpha|100\rangle - \beta|101\rangle - \beta|110\rangle + \alpha|111\rangle}{2} = \pi_2$$

This is the state after the $CNOT$ and the Hadamard. We express this as a state vector:

$$\frac{1}{2} \cdot \begin{pmatrix} \alpha \\ \beta \\ \beta \\ \alpha \\ \alpha \\ -\beta \\ -\beta \\ \alpha \end{pmatrix}$$

We measure the first qubit m_1.

Conceptional view:

$$\left(\frac{1}{2}\right)\cdot(\alpha|000\rangle+\beta|001\rangle+\beta|010\rangle+\alpha|011\rangle+\alpha|100\rangle-\beta|101\rangle-\beta|110\rangle+\alpha|111\rangle)=$$

$$\left(\frac{1}{2}\right)\cdot\big(|0\rangle(\alpha|00\rangle+\beta|01\rangle+\beta|10\rangle+\alpha|11\rangle)+|1\rangle(\alpha|00\rangle-\beta|01\rangle-\beta|10\rangle+\alpha|11\rangle)\big)$$

State vector view:

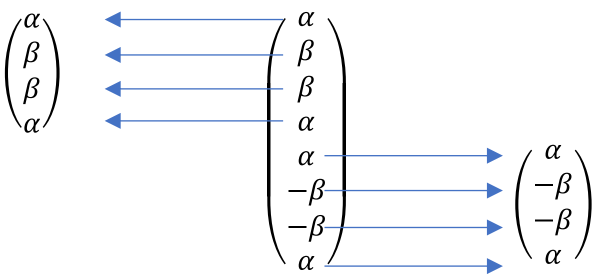
$$\frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix}\otimes\left(\alpha\begin{pmatrix}1\\0\\0\\0\end{pmatrix}+\beta\begin{pmatrix}0\\1\\0\\0\end{pmatrix}+\beta\begin{pmatrix}0\\0\\1\\0\end{pmatrix}+\alpha\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right)+\frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix}\otimes\left(\alpha\begin{pmatrix}1\\0\\0\\0\end{pmatrix}-\beta\begin{pmatrix}0\\1\\0\\0\end{pmatrix}-\beta\begin{pmatrix}0\\0\\1\\0\end{pmatrix}+\alpha\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right)=$$

$$\frac{1}{2}\cdot\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}\alpha\\\beta\\\beta\\\alpha\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}\alpha\\-\beta\\-\beta\\\alpha\end{pmatrix}\right)$$

What has happened: The 8D state vector collapses into two possible 4D state vectors:

$$\begin{pmatrix}\alpha\\\beta\\\beta\\\alpha\end{pmatrix}\text{ or }\begin{pmatrix}\alpha\\-\beta\\-\beta\\\alpha\end{pmatrix}$$

Measuring in the standard basis splits the state vector:



After the measurement we get either the state $\begin{pmatrix}\alpha\\\beta\\\beta\\\alpha\end{pmatrix}$ or the state $\begin{pmatrix}\alpha\\-\beta\\-\beta\\\alpha\end{pmatrix}$.

We measure the second qubit. As the 8D state vector has two possible outcomes to become a 4D state vector, we have two paths to follow.

In the conceptual view:

$\left(\frac{1}{2}\right)\cdot(\alpha\|00\rangle+\beta\|01\rangle+\beta\|10\rangle+\alpha\|11\rangle)=$ $\left(\frac{1}{2}\right)\cdot\big(\|0\rangle(\alpha\|0\rangle+\beta\|1\rangle)+\|1\rangle(\beta\|0\rangle+\alpha\|1\rangle)\big)$	$\left(\frac{1}{2}\right)\cdot(\alpha\|00\rangle-\beta\|01\rangle-\beta\|10\rangle+\alpha\|11\rangle)=$ $\left(\frac{1}{2}\right)\cdot\big(\|0\rangle(\alpha\|0\rangle-\beta\|1\rangle)+\|1\rangle(-\beta\|0\rangle+\alpha\|1\rangle)\big)$

In case we measure $|0\rangle$ for the first qubit we get either $\alpha|0\rangle+\beta|1\rangle$ or $\alpha|0\rangle-\beta|1\rangle$ for the second qubit.

In case we measure $|1\rangle$ for the first qubit we get either $\beta|0\rangle + \alpha|1\rangle$ or $-\beta|0\rangle + \alpha|1\rangle$ for the second qubit.

In state vector view:

$\left(\frac{1}{2}\right) \cdot \left(|0\rangle(\alpha|0\rangle + \beta|1\rangle) + |1\rangle(\beta|0\rangle + \alpha|1\rangle)\right) \rightarrow$ $\frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix}\right) + \frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix} \otimes \left(\beta\begin{pmatrix}1\\0\end{pmatrix} + \alpha\begin{pmatrix}0\\1\end{pmatrix}\right)$	$\left(\frac{1}{2}\right) \cdot \left(|0\rangle(\alpha|0\rangle - \beta|1\rangle) + |1\rangle(-\beta|0\rangle + \alpha|1\rangle)\right) \rightarrow$ $\frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\alpha\begin{pmatrix}1\\0\end{pmatrix} - \beta\begin{pmatrix}0\\1\end{pmatrix}\right) + \frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix} \otimes \left(-\beta\begin{pmatrix}1\\0\end{pmatrix} + \alpha\begin{pmatrix}0\\1\end{pmatrix}\right)$

The 4D state vector collapses with equal probability to:

$\frac{1}{2}\begin{pmatrix}\alpha\\\beta\end{pmatrix}$ or $\frac{1}{2}\begin{pmatrix}\beta\\\alpha\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}\alpha\\-\beta\end{pmatrix}$ or $\frac{1}{2}\begin{pmatrix}-\beta\\\alpha\end{pmatrix}$

In detail:

<table>
<tr><td colspan="2">First measurement gives $\begin{pmatrix}1\\0\end{pmatrix}$,
system collapses to $\begin{pmatrix}\alpha\\\beta\\\beta\\\alpha\end{pmatrix}$</td><td colspan="2">First measurement gives $\begin{pmatrix}0\\1\end{pmatrix}$,
system collapses to $\begin{pmatrix}\alpha\\-\beta\\-\beta\\\alpha\end{pmatrix}$</td></tr>
<tr><td>Second measurement gives $\begin{pmatrix}1\\0\end{pmatrix}$,
system collapses to $\begin{pmatrix}\alpha\\\beta\end{pmatrix}$.</td><td>Second measurement gives $\begin{pmatrix}0\\1\end{pmatrix}$,
system collapses to $\begin{pmatrix}\beta\\\alpha\end{pmatrix}$.</td><td>Second measurement gives $\begin{pmatrix}1\\0\end{pmatrix}$,
system collapses to $\begin{pmatrix}\alpha\\-\beta\end{pmatrix}$.</td><td>Second measurement gives $\begin{pmatrix}0\\1\end{pmatrix}$,
system collapses to $\begin{pmatrix}-\beta\\\alpha\end{pmatrix}$.</td></tr>
</table>

From this we learn:

- If the result of the measurements is $\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$, then the remaining state vector is $\begin{pmatrix}\alpha\\\beta\end{pmatrix}$.
- If the result of the measurements is $\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$, then the remaining state vector is $\begin{pmatrix}\beta\\\alpha\end{pmatrix}$.
- If the result of the measurements is $\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$, then the remaining state vector is $\begin{pmatrix}\alpha\\-\beta\end{pmatrix}$
- If the result of the measurements is $\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$, then the remaining state vector is $\begin{pmatrix}-\beta\\\alpha\end{pmatrix}$

In the first case we need not do anything, the remaining qubit is the one Alice had.

In the second case we need to flip the qubit.

In the third case we need a phase shift on the second entry.

In the fourth case we need both, a flip and a phase shift on the second entry.

We see that Bob needs information about the result of the measurement to be able to restore Alice's qubit correctly.